LOYOLA COLLEGE (AUTONOMOUS), CHENNAI – 600 034

M.Sc. DEGREE EXAMINATION - **PHYSICS**

SECOND SEMESTER – **NOVEMBER 2013**

PH 2812 - MATHEMATICAL PHYSICS

Date : 15/11/2013 Dept. No. Max. : 100 Marks
Time : 9:00 - 12:00

PART - A

Answer **ALL** questions (10x2=20)

1) Evaluate the complex line integral $\int \frac{dz}{z}$ around the closed loop C: |z| = 1.

2) Determine the residue at Z= 0 and at Z = i of the complex function f (z) = $\frac{9Z+i}{Z(Z^2+1)}$.

3) Define Dirac delta function. What is its Laplace transform?

4) Define the unit step function (Heaviside function) and find its Laplace transform.

5) What are the two possible initial conditions in the vibration of a rectangular membrane? Explain the symbols used.

6) Solve $\frac{\partial^2 u(x,y)}{\partial x^2} - u(x,y) = 0$.

7) Use the Rodrigue's formula to evaluate the 3rd degree Legendre polynomial .

8) State the orthonormality property of the Hermite polynomials.

9) List the four properties that are required by a group multiplication.

10) Distinguish an Abelian group from a cyclic group.

PART - B

Answer any **FOUR** questions (4 x 7.5 = 30)

11) Determine whether the function v = e^x sin y is harmonic. If your answer is yes, find a corresponding analytic function f (z) = u (x, y) + i v (x, y). (3+4.5)

12) Solve the initial value problem $\frac{d^2y}{dt^2} + 25y = 10\cos 5t$, y (0) = 2, $\frac{dy(0)}{dt} = 0$ using Laplace transform technique .

13) Use the method of separation of variables to solve the partial differential equation $\frac{\partial u}{\partial x} = 2\frac{\partial u}{\partial t} + u$, where u(x,0) = 6 e^{-3x}.

14) (a) Prove that $J_{-n}(x) = (-1)^n J_n(x)$ if n is a positive integer where $J_n(x)$ is the Bessel function of first kind.

(b) Determine the value of $J_{-1/2}(x)$. (5+2.5)

15) Work out the multiplication table of the symmetry group of the proper covering operations of an equilateral triangle. Write down all the subgroups and divide the group elements into classes. What are the allowed dimensionalities of the representation matrices of the group?

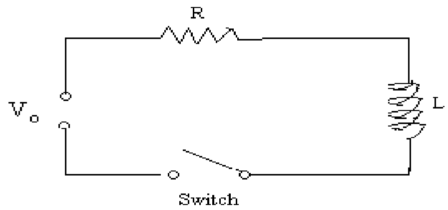
PART - C

Answer any **FOUR** questions (4 x 12.5 = 50)

16) (a) Using the contour integration, evaluate the real integral, $\int_0^\infty \frac{dx}{(1+x^2)^3}$. (6.5)

(b) Evaluate the following integral using Cauchy's integral formula $\int_c \frac{4-3z}{z(z-1)(z-2)}$ dz, where C is the circle |Z |= 3/2. (6)

17) Find the current i(t) in the LC circuit shown in figure by setting up the differential equation for the problem and solving it by Laplace transforms . Assume zero initial current and charge on the



capacitor and V_o , a constant voltage.

18) Solve the one- dimensional wave equation $\frac{\partial^2 u}{\partial t^2} = c^2 \frac{\partial^2 u}{\partial x^2}$ by the separation of variable technique and the use of Fourier series. The boundary conditions are u(0,t) =0 and u(L,t)=0 for all t and the initial conditions are u (x,0) = f(x) and ∂u/∂t = g(x) at t =0. (Assume that u (x,t) to represent the deflection ofstretched string and the string is fixed at the ends x = 0 and x = L).

19) (a) Solve the Legendre differential equation $(1 - x^2) \frac{d^2y}{dx^2} - 2x \frac{dy}{dx} + n(n+1)y = 0$ by the power series method. (6.5)

(b) Establish the orthonormality relation $\int_{-1}^{+1} P_n(x) P_m(x)\, dx = \frac{2}{(2n+1)} \delta_{nm}$ where $P_n(x)$ is the Legendre polynomial of order n. (6)

20) (a) Prove that any representation by matrices with non-vanishing determinants is equivalent to a representation by unitary matrices. (6.5)

(b)Enumerate and explain the symmetry elements of CO_2, H_2O and NH_3 molecules. (6)

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